CSCI 432 Handout 14: Minimum Spanning Trees

Name: _____

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Examples

1. Draw an example of a weighted graph with two distinct minimum spanning trees.

   Answer

2. What assumptions will remove this issue?

   Answer

3. For the next two questions, suppose we have a weighted graph $G = (V, E, \omega)$ meeting the asusmptions listed above whose minimum spanning tree is T. Let F be a subgraph of T.

   If $e \in E \setminus F$ and both endpoints of e are in the same component of F, is $e \in T$? Why or why not?

   Answer

4. Let $E' \in E \setminus F$ be the set of edges such that their endpoints are in different components of F. Let e be the minimum-weight edge in E'. Is $e \in T$? Why or why not?

   Answer

5. What assumptions will remove this issue?

   Answer

Borůvka's Algorithm

In the textbook, Dr. Erickson describes a generic MST algorithm: We start with F as the set of vertices (a spanning forest, if you will), and add edges to R until we find a MST.

In Borůvka's algorithm, all safe edges are added at each step.

1. Walk through an example, marking all safe edges and all useless edges at each step. (The graph you start with should have at least eight vertices and 10 edges, and should satisfy the assumptions we listed on the first page).

   Answer

2. What is an observation you have about the useless edges?

   Answer

3. What is the loop invariant for this algorithm?

   Answer

Jarnik's / Prim's Algorithm

In Jarnik's algorithm, we have a subtree T' of the MST T'. We obtain this by starting at an arbitrary vertex and adding the minimum safe edge for T' at each step.

1. Walk through an example. (The graph you start with should have at least eight vertices and 10 edges, and should satisfy the assumptions we listed on the first page).

   Answer

2. What is the loop invariant for this algorithm?
   Answer

3. What are some pros/cons of using Jarnik's versus Borůvka's algorithms?
   Answer

Kruskal's Algorithm

In Kruskal's algorithm, we again have a spanning forest F at each stage, initialized to each vertex is its own tree. At each step, we pick the minimum weight edge and add it to F, as long as it is not useless.

1. Walk through an example. (The graph you start with should have at least eight vertices and 10 edges,

and should satisfy the assumptions we listed on the first page).

Answer