CSCI 432 Handout 06: Master Theorem

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Master's Theorem

Master's theorem allows us to quickly solve recurrence relations of the form:

$$T(n) = aT(n/b) + f(n),$$

where $a, b \in \mathbb{N}$ such that $a \geq 1$ and $b > 0$ and $f(n)$ is asymptotically positive. Then, we can determine the closed-form of $T(n)$ as follows:

1. IF there exists $\varepsilon \in \mathbb{R}_+$ such that $f(n) \in O(n^{\log_b a - \varepsilon})$, THEN $T(n) \in \Theta(n^{\log_b a})$.
2. IF there exists $\varepsilon \in \mathbb{R}_+$ such that $f(n) \in \Theta(n^{\log_b a})$, THEN $T(n) \in \Theta(n^{\log_b a} \log n)$.
3. IF
   (a) there exists $\varepsilon \in \mathbb{R}_+$ such that $f(n) \in \Omega(n^{\log_b a + \varepsilon})$ and
   (b) there exists $c \in (0, 1)$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, the following holds: $af(n/b) \leq cf(n)$,

   THEN $T(n) \in \Theta(f(n))$.

	a	b	$\log_b a$	$n^{\log_b a}$	$f(n)$	Potential Case?	ε, if Case 1 or 3	Closed Form
$T(n) = T(n/2) + 1$								
$T(n) = 2T(n/4) + \sqrt{n}$								
$T(n) = 2T(n/4) + n$								
$T(n) = 2T(n/4) + n^2$								
$T(n) = 3T(n/3) + \Theta(1)$								

Remember, Case 3 has an additional condition to check (this condition is called the *regularity condition*)! Do that in the space provided below, or on the back of this page.

More Recurrence Relations

What is the asymptotic form of the following recurrence relations? Let $T\colon \mathbb{N} \to N$ be defined by $T(1) = 1$ and, for $n > 1$,

1. $T(n) = 16T(n/4) + n$
   Try it!

2. $T(n) = 2T(n/2) + n \log n$
   Try it!

3. $T(n) = 6T(n/3) + n^2 \log n$
   Try it!

4. $T(n) = 4T(n/2) + n^2$
   Try it!

5. $T(n) = 9T(n/3) + n$
   Try it!

6. $T(n) = 3T(n/4) + n \log n$
   Try it!

7. $T(n) = 2T(n/3) + 2T(n/4) + n^2$
   Try it!