CSCI 432 Handout 04: Recurrence Relations

Name: ____________________

Collaborators: ____________________

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Recall a Couple Common Summations

There are a couple summations that we see over and over again in algorithms. Let's see which you remember. Please fill out the following table, where all functions are of the form $f_i\colon \mathbb{N} \to \mathbb{R}$, and $c \in \mathbb{R}$ is a constant.

Name	Summation	Closed Form	Asymptotic Form
Arithmetic Series	$f_1(n) = \sum_{i=0}^{n} c$		
Constant Summation	$f_2(n) = \sum_{i=0}^{n} i$		
Sum of Squares	$f_3(n) = \sum_{i=1}^{n} i^2$		
Finite Geom. Series	$f_4(n) = \sum_{i=0}^{n} cr^i$	$f_4(n) = \begin{cases} c\frac{1-r^{n+1}}{1-r} & r \neq 1 \\ ? & r = 1 \end{cases}$	
Harmonic Series	$f_5(n) = \sum_{i=1}^{n} \frac{1}{i}$	(no nice closed form)	

Summations and Recursions

Now, let's write each of the summations above as a recursion by 'plucking off' the first term. For example,

$$f_1(n) = \begin{cases} c & n = 1 \\ c + f_1(n-1) & otw \end{cases}$$

1. $f_2(n) =$

2. $f_3(n) =$

3. $f_4(n) =$

4. $f_5(n) =$

Being able to do this is helpful for:

- In the inductive step of a proof by induction.
- To walk through algorithms.

Recursion

For each of the recursions below, name an algorithm that uses that recursion, and give the asymptotic form of the recurrence relation.

1. $T(n) = 2T(n/2) + \Theta(n)$
   Answer:

2. $T(n) = T(n/2) + \Theta(1)$
   Answer:

3. $T(n) = T(n-1) + \Theta(1)$
   Answer:

4. $T(n) = T(n-1) + \Theta(n)$
   Answer:

5. $T(n) = 2T(n-1) + \Theta(1)$
   Answer:

6. $T(n) = T(n-1) + T(n-2) + \Theta(1)$
   Answer:

We often dived the problem in half. What happens in the following two recursions if we divide the problem into thirds?

1. $T(n) = 3T(n/3) + \Theta(n)$
   Answer:

2. $T(n) = T(n/3) + \Theta(1)$
   Answer: