5 Feb 2026

Pseudocode + proofs

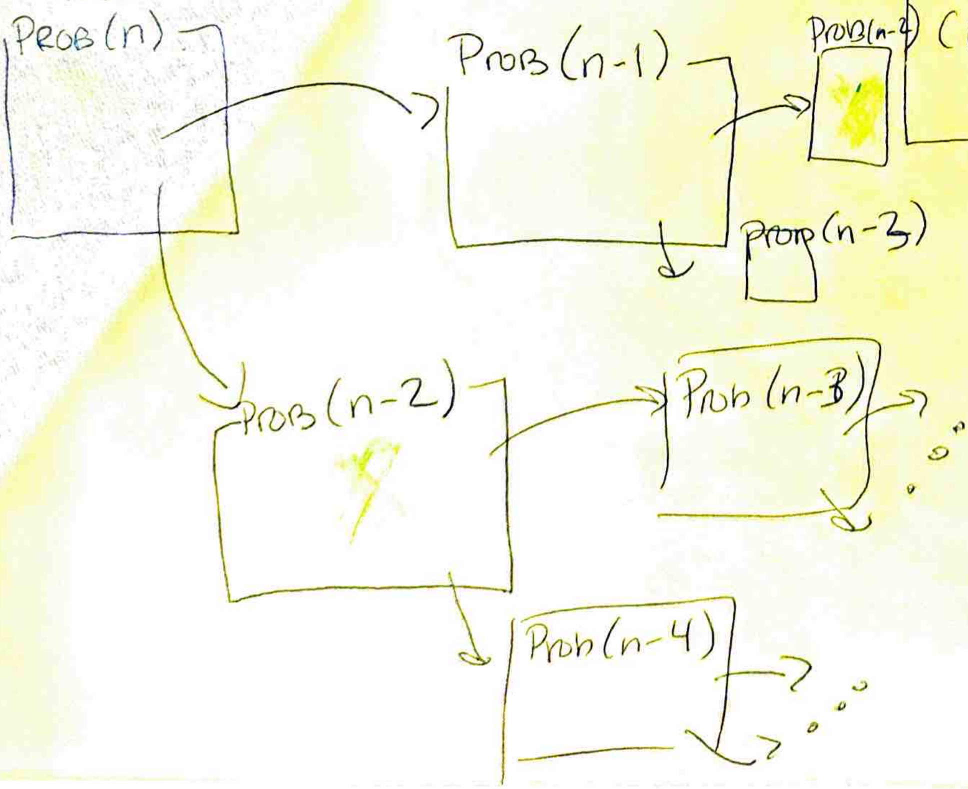
beware of pharases like "and so on"
or arguments that use "..."

→ hides a loop or recursion

```
InspectDoll (n)
  check painting*
  if n=1  (doesnt open)
  | return Good
  else if n>1 (it opens)
  |   open doll
  |  x←Inspect Doll (n-1)
  |   check both halves *
  end if
  return X
```

*can return "BAD" if something goes wrong

what recursive calls look like for FIB:



Stack overflow can be a problem w/ recursion

For FIB, would be better to create a table + store partial results

↑

Dynamic Programming (smart memoization)

regularity condition

need: $\exists\, c \in (0,1)$ and $n_0 \in \mathbb{N}$ s.t. $a f(n/b) \leq c \cdot f(n)$

↑ open interval

For problem c:
$$a f(n/b) \leq c f(n)$$
$$\Updownarrow$$
$$2 f(n/4) \leq c f(n)$$ by plugging in a, b

~~plugging in~~ use def'n $f(n) = n$:

$$n/2 = 2(n/4) \leq c \cdot n$$

choose $c = 1/2$, $n_0 = 5$

$\forall n \geq 5$, $n/2 \leq \frac{1}{2} n$ Yup, that's true!

(I don't even need an induction to prove this.)

Closed form
$$T(n) \in \Theta(f(n)) = \Theta(n)$$

CSCI 432 Handout 06: Master Theorem

Name: ______________________________

Collaborators: ______________________________

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my prob. of size n can be broken up into $\boxed{a}$ problems, $\uparrow$ a # each of size (n/b) and it takes $f(n)$ time to break it up & put it back together

Master's Theorem

Master's theorem allows us to quickly solve recurrence relations of the form:

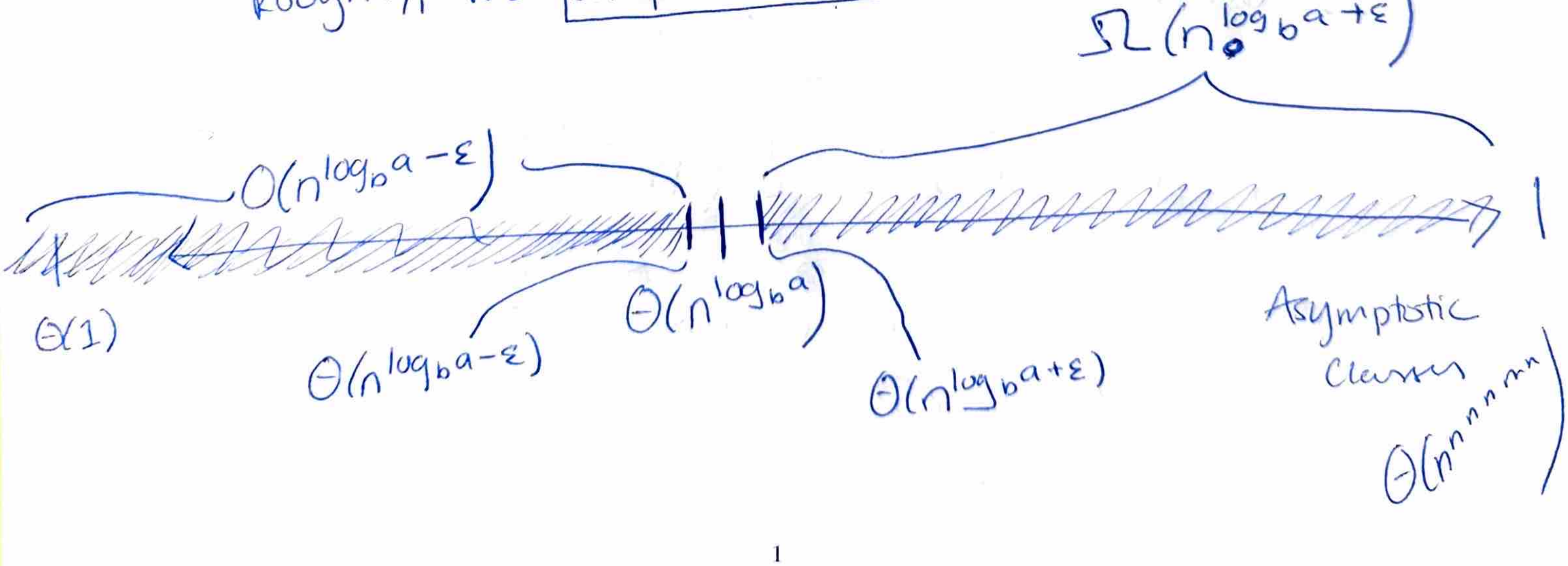
$$\boxed{T(n) = aT(n/b) + f(n),}$$

where $a, b \in \mathbb{N}$ such that $a \geq 1$ and $b > 0$ and $f(n)$ is asymptotically positive. Then, we can determine the closed-form of $T(n)$ as follows:

1. IF there exists $\varepsilon \in \mathbb{R}_+$ such that $\boxed{f(n) \in O(n^{\log_b a - \varepsilon})}$, THEN $T(n) \in \Theta(n^{\log_b a})$. *
2. IF there exists $\varepsilon \in \mathbb{R}_+$ such that $\boxed{f(n) \in \Theta(n^{\log_b a})}$, THEN $T(n) \in \Theta(n^{\log_b a} \log n)$. *
3. IF
   (a) there exists $\varepsilon \in \mathbb{R}_+$ such that $\boxed{f(n) \in \Omega(n^{\log_b a + \varepsilon})}$ and *
   (b) there exists $c \in (0, 1)$ and $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, the following holds: $af(n/b) \leq cf(n)$,

   THEN $T(n) \in \Theta(f(n))$.

Roughly, we $\boxed{\text{compare } f(n) \text{ to } \Theta(n^{\log_b a})}$ *

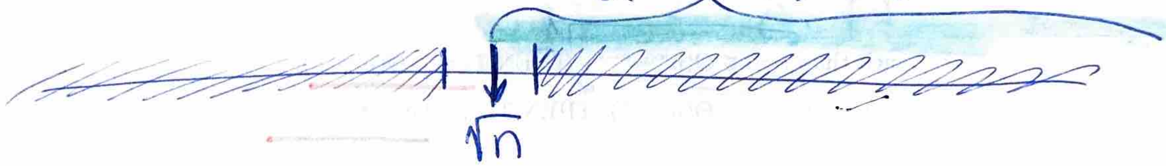


		1	2	3	4	5	6	7	8
		a	b	$\log_b a$	$n^{\log_b a}$	$f(n)$	Potential Case?	ϵ, if Case 1 or 3	Closed Form
A	$T(n) = T(n/2) + 1$								
B	$T(n) = 2T(n/4) + \sqrt{n}$								
C	$T(n) = 2T(n/4) + n$	2	4	$\log_4 2 = 1/2$	$n^{1/2} = \sqrt{n}$	$f(n)=n \in \Theta(n)$	3	$\varepsilon = 0.25$, $c = 0.5$	$\Theta(n)$
D	$T(n) = 2T(n/4) + n^2$								
E	$T(n) = 3T(n/3) + \Theta(1)$					$f(n)=6$, $f(n)=3c$ ↑ PICK any.			

Remember, Case 3 has an additional condition to check (this condition is called the *regularity condition*)! Do that in the space provided below, or on the back of this page.

$\Theta(f(n)=n) > \Theta(\log_b a = \sqrt{n})$

C:



Potential case is Case 3.
Question is: Is it in the shaded region, or does it fall in the gap?
Is there a ε such that

$$f(n) \in \Omega(n^{\log_b a + \varepsilon})$$

I.O.W, for this problem: $[f(n) = n^1] \in \Omega(n^{1/2+\varepsilon})$

notes: $\varepsilon = 0$ OK ← ~~neither~~ these not allowed as ε

$\varepsilon = 1$ not OK

Try: $\varepsilon = 0.25$

$$[f(n) = n^1] \overset{?}{\in} \Omega(n^{0.5+0.25})$$

$$\in \Omega(n^{0.75})$$ YES!

$\varepsilon = 0.5$ makes this tight $f(n) \in \Theta(n^{0.5+0.5})$

[must check the regularity condition]

5. $T(n) = 9T(n/3) + n$
Try it!

6. $T(n) = 3T(n/4) + n \log n$
Try it!

7. $T(n) = 2T(n/3) + 2T(n/4) + n^2$ $\leq 4T(n/3) + n^2 \rightarrow T(n) \in O(-)$
Try it!

$T(n) \geq 4T(n/4) + n^2$
$\hookrightarrow \ldots T(n) \in \Omega(-)$

maybe try both of these to get upper/lower bounds