$\forall n \geq 1$,

Claim: A path with n vertices has $n-1$ edges.

[set-up]

Proof:

We proceed by induction.

For the base case, we have $n=1$.

1 vertex, 0 edges ✓

Let $k \geq 1$, assume that any ~~graph~~ path with k vertices has $k-1$ edges.

Inductive step:

((WTS: path with $k+1$ verts has k edges)

Let P be a path with $k+1$ vertices.

This is "generalizing from the general particular."

Let v_0 be the 1st vertex in P.

So, $P = [v_0, v_1, v_2 \ldots, v_k]$.

Consider $P' = [v_1, v_2, \ldots, v_k]$

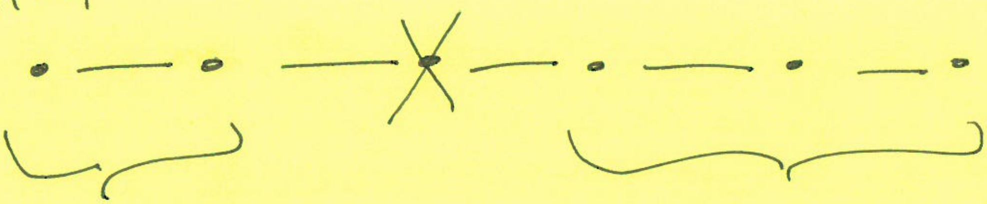
removed v_0 and edge (v_0, v_1)

By I.A., P' has $k-1$ edges. To go "backwards", re-add v_0 and (v_0, v_1) to P' to re-obtain P, which gives us $k+\underbrace{1}_{v_0}$ vertices and $\underbrace{k-1}_{\text{from } P} + \underbrace{1}_{(v_0, v_1)} = k$ edges.

Note: with strong ind., can remove any vertex.

$k=5$

graph w/ $k+1$ vertices



2 verts

3 verts

∴ needs strong induction