Greedy
CSCI 432

Greedy Algorithms:

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- Do not look ahead, plan, or revisit past decisions.
- Hope that optimal local choices lead to optimal global solutions.

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Greedy algorithm for:
- Robbing a jewelry store?
- Eating at a fancy buffet?
- Triaging buddy after failed cornice huck?

Activity Selection

Input:
- $S = \{a_1, a_2, \ldots, a_n\}$ – set of courses that need rooms.
- $a_i = (s_i, f_i)$ – start and finish times for each course.

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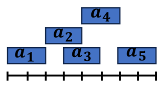

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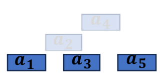

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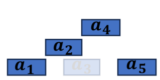

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Greedy selection criteria?

Algorithm Outline?

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Greedy decision?
Smallest duration first.

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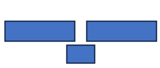

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Smallest conflict first.

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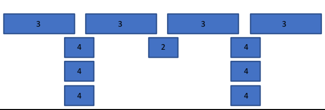

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Goal: Select a maximum sized subset of mutually compatible courses.

Greedy decision?
Earliest compatible finish time first.

Activity Selection

```
activity_selection(activities A)
  sort_by_finish(A)
  selected = {A[1]}
  last_added = 1
  for i = 2 to A.length
    if A[i].start ≥ A[last_added].finish
      selected  = selected  ∪ {A[i]}
      last_added = i
  return selected
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Running Time?

Validity?

Performance?

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Running Time? $O(n \log n)$

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Validity? `selected` consists of compatible courses.

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Validity? `selected` consists of compatible courses.

Performance? Is `selected` the largest possible subset?

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<u>Greedy decision:</u> Select the next course with the earliest compatible finish time.

<u>Proof of optimality:</u>

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Proof of optimality: Let A be the set of courses, $S \subseteq A$ be the greedy algorithm selection, and $OPT \subseteq A$ be an optimal selection, all sorted by increasing finish time.

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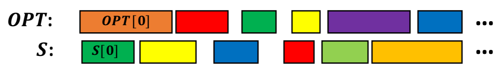

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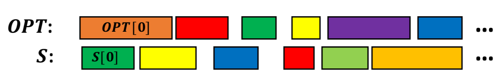

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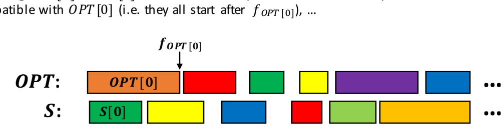

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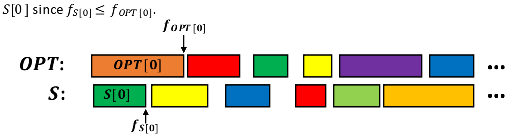

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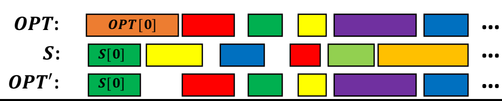

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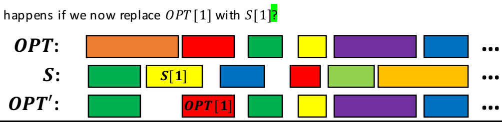

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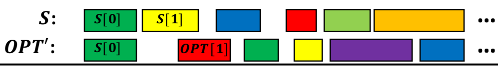

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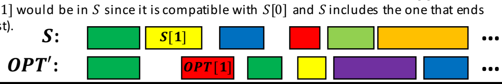

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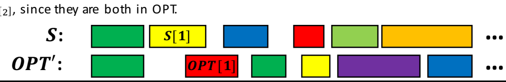

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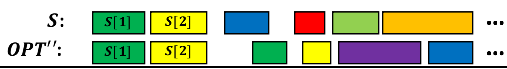

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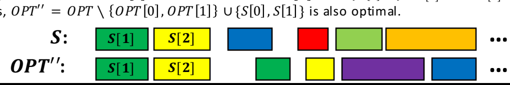

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We can then proceed inductively and show that each course in OPT can be replaced by the corresponding course in S without violating compatibility restrictions. i.e., We translated OPT into S at no extra cost, thus S must be optimal.