20 Sept 2023

First pass of Quick select,
the worst-case recurrence relation was:

$$T(n) = T(n-1) + \Theta(n)$$

$$\Rightarrow T(n) \in \Theta(n^2).$$

What if I can guarantee my partition element "shaves off" a %-age each time?
Then, recursion would be

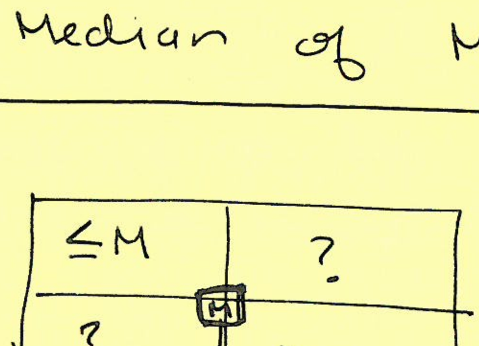
$$T(n) = T(n/b) + \Theta(n)$$

$$\Rightarrow T(n) \in \Theta(n \log n)$$ by Master's Theor

So, we will change line 4 from:

4: Choose a pivot $p \in \underline{n}$

to the "Median of Medians" approach



At worst, every elt in ? regions in the size we recurse on

M = median of medians

Median of Medians approach

Have: A = an array of length n, unsorted

Want: central-ish pivot
(at least $\Theta(n/b)$ are on Both sides of it

$\uparrow$ $\in \mathbb{R}$, ≥ 1

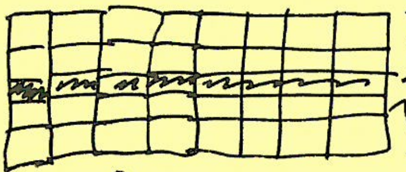
How?

$\Theta(n)$ ① Divide A into $k = \lceil n/5 \rceil$ groups of size 5.

$k \cdot \Theta(1)$ ② For each group, calculate the median m_i

③ Find the median by calling

$T(k)$ QUICK SELECT $\left(\{m_i\}_{i=1}^{k}, k/2\right)$



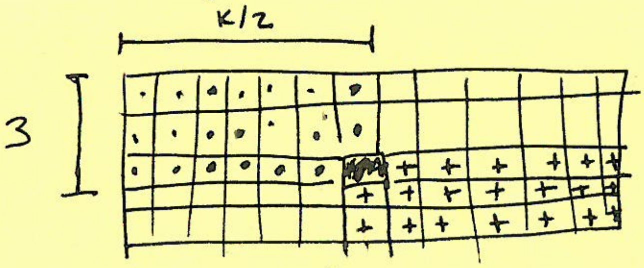
} less than ▨ directly below it

} bigger than ▨ directly above it

ith group is a column of values

Imagine: re-arranged so sorted by median element

Then, I know:



smallest med → largest median

every $\boxed{+}$ is bigger than my median of medians

▨ $\geq$ $\boxed{\cdot}$

And there are

$\left(\frac{k}{2}\right) \cdot 3 = \left(\frac{n/5}{2} \cdot 3\right)$

$= \frac{3n}{10} - 1$ elem

$\Rightarrow$ there are $\Theta(\frac{7}{10}$

elements in $\square$

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Lets' Revisit The recurrence relation (using line #s from Monday)

$T(n) = \Theta(1)$, lines 1-3

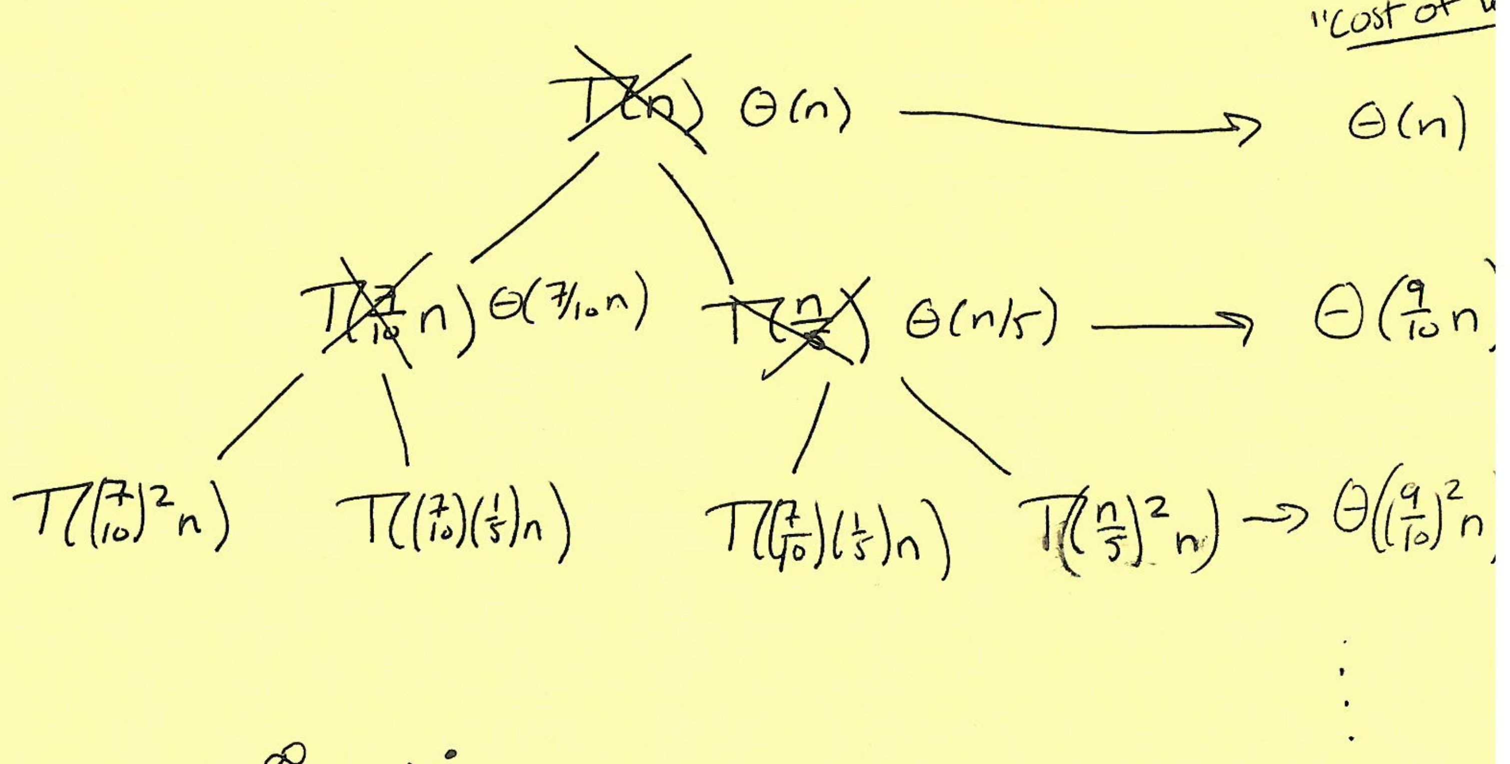
$+ \Theta(n) + \Theta(n/5) + T(n/5)$, new line 4

$+ T(\frac{7}{10} n)$, new worst-case recurrence in lines 8-12

$= T(\frac{7}{10}n) + T(\frac{n}{5}) + \Theta(n)$

But wait! Now there are 2 recurrence relations

Let's look at that recursion tree



$T(n) \leq c \cdot n \sum_{i=0}^{\infty} (\frac{9}{10})^i$, a geometric series

l^{th} level: $\Theta((\frac{9}{10})^l n)$

$= c \cdot n \cdot 10 \in \Theta(n)$ $\Rightarrow$ we have a worst-case linear time selection!

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What if ... we used groups of size 3 instead?

① What is the recurrence relation?

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + \Theta(n)$$

② What is the asymptotics of that RR?

use MT on the following:

$$T(n) \leq 2T\left(\frac{2n}{3}\right) + \Theta(n)$$

$$T(n) \geq 2T\left(\frac{n}{3}\right) + \Theta(n)$$