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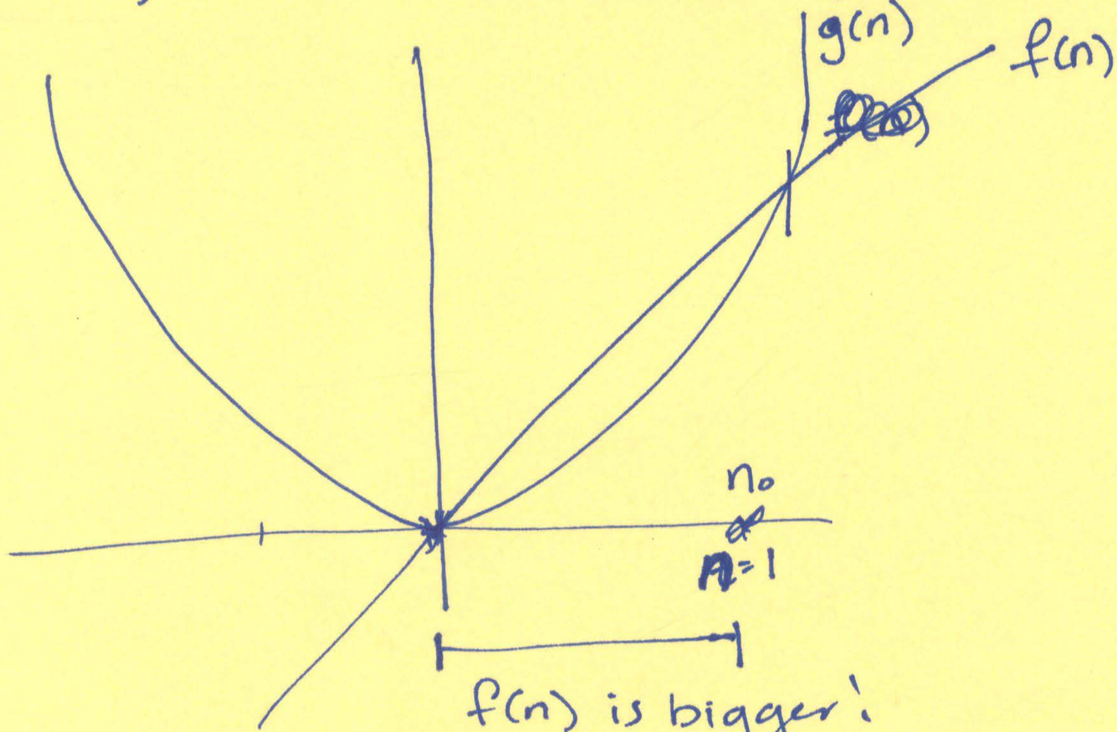
Real-RAM model of computation

- Real = real numbers are what we assume we can store

- RAM = random access machine
  - ↳ given a location in memory, can return what is stored there in constant time (1 unit of time)

- things we can do in constant time:
  - → access/read a piece of memory
  - → write a piece of memory
  - → basic math operations: $+$, $-$, $\times$, $\div$, log

Comparing $f(n)=n$ and $g(n)=n^2$



CSCI 432 Handout 03: Asymptotic Notation

Name(s): CLASS

30 August 2023

$\mathbb{Z}_+ = \{1, 2, 3, \ldots\}$
$\mathbb{N} = \{0, 1, 2, \ldots, \ldots\}$

Definitions

First, let's recall the definitions:

initial point

big-O

Definition 1 (Asymptotic Notation). *Let $f, g\colon \mathbb{N} \to \mathbb{R}$. Then, we say that $\boxed{f \text{ is } O(g)}$ iff: there exists $n_0 \in \mathbb{N}$ and $\underbrace{c}_{\text{constant}} \in \mathbb{R}_+$ such that for all $n \geq n_0$, the following holds:*

$$0 \leq f(n) \leq cg(n).$$

omega

This is interpreted as "f is upper-bounded by g." In addition, if those same conditions hold, we also say that $\boxed{g \text{ is } \Omega(f).}$ This is interpreted as "~~f~~ g is lower-bounded by f."

theta

The asymptotic tight bound is denoted by Θ. For $f, g\colon \mathbb{N} \to \mathbb{R}$, we say that f is $\Theta(g)$ iff f is $O(g)$ and f is $\Omega(g)$. This is the bound that we most often want to find!

Let's practice! We will complete some of the following problems in class. What we do not complete, please do them on your own. These will be collected for attendance, but not graded. Nonetheless, but you are expected to know how to answer these questions. If you have any questions, please reach out to the instructor for help.

1. Let $f\colon \mathbb{N} \to \mathbb{R}$ be defined by $f(n) = n^2 + 2$. Prove that $f(n)$ is $O(n^2)$.
   Try it!

WTS: $0 \leq n^2 + 2 \leq c \cdot n^2$ ← need to find c and an initial n_0 that works.
✓

Try: $\boxed{c = 2}$ b/c 2 terms each of which is $\leq n^2$

$n^2 \leq n^2$ ✓

$2 \leq n^2$

$\Rightarrow \boxed{n_0 \geq 2}$

NOW, we use induction to prove this.

note: n_0 doesn't work!

Scratch work:

$(k+1)^2-2 \le (k+1)^2$

$k^2+2k+1-2 \le k^2+2k+1$

2. Let $f\colon \mathbb{N} \to \mathbb{R}$ be defined by $f(n) = n^2 - 2$. Prove that $f(n)$ is $O(n^2)$.
   Try it!

I think $c = 1$ and $n_0 = 0$ works.

Base: TODO

I.A: Let $k \ge 0$. Assume $0 \le k^2 - 2 \le k^2$

Inductive step: (WTS: $0 \le (k+1)^2 - 2 \le (k+1)^2$)

$(k+1)^2 - 2 = k^2 + 2k + 1 - 2$

$= k^2 + 2k - 1$

$\le k^2 + 2k + 1 = (k+1)^2 \quad \therefore (k+1)^2 - 2 \le (k+1)^2$

3. If f is $\Theta(g)$, is it true or false that g is $\Theta(f)$? Why or why not?
   Try it!

4. Prove that $f\colon \mathbb{N} \to \mathbb{R}$ defined by $f(n) = \log_2(n)$ is $\Theta(\log_{10}(n))$.
   Try it!

5. Prove that Θ determines an equivalence relation on functions.
   Try it!