INDUCTION EXAMPLE: 28 Aug 2023

Claim: The complete graph on n vertices has $\frac{n(n-1)}{2}$ edges.

Proof: We prove this claim by induction.

For the base case, let $n = 1$.

mini proof.

The complete graph on 1 vertex is $\bullet$ } 1 vertex, 0 edges.

Note $\frac{n(n-1)}{2} = \frac{1(1-1)}{2} = \frac{1(0)}{2} = 0$ ✓

$\boxed{\begin{array}{l}\text{Let } k \geq 1. \\ \text{We assume the complete graph on } k \\ \text{vertices has } \frac{k(k-1)}{2} \text{ edges.}\end{array}}$

our I.A.

WTS: the complete graph on $k+1$ vertices has $\frac{k(k-1)}{2}$ edges.

Let K_{k+1} be the complet graph on $k+1$ verts.

Let v be a vertex in K_{k+1}. Removing it & it's ~~vertices~~ k edges results in the complete graph on k vertices, $K_k \subset K_{k+1}$.

That subgraph, by our IA, has $\frac{k(k-1)}{2}$ edges.

Adding our removed edges, K_{k+1} has

$$\frac{k(k+1)}{2} + k = \frac{k(k+1)}{2} + \frac{2k}{2} = \frac{k(K+1+2)}{2} = \frac{k(k+1)}{2}$$